國立中央大學九十學年度轉學生入學試題

物理學系 三年級 科目： 古典物理 共 1 頁 第 1 頁

(25%) 1. Consider the motion of a particle of mass m in a central-force field $\vec{F}(\vec{r}) = \hat{r}F(r)$, where $\hat{r} = \vec{r}/r$, and $\vec{r}$ is the position vector of the particle relative to the center of the force.

(1). Show that the force is a conservative force.

(2). Show that the orbital angular momentum of the particle $\vec{L}$ about the center of force is a constant of motion. Thus both the position vector and the linear momentum vector of the particle lie always in a plane normal to $\vec{L}$, and we have a two-dimensional problem.

(3). Show that if $F(r) = \frac{-k}{r^2}$, where k is a positive constant, then the path of the particle is a conic section.

(25%) 2. Two equal masses m are equally spaced along a massless string of length $3a$. The free ends of the string are rigidly fixed. The tension in the string is τ. Find the two normal mode angular frequencies and the corresponding normal mode displacements for small transverse oscillations of the masses. Neglect the gravitational effect.

(25%) 3. (1). Write down the differential form of Maxwell's equations for electromagnetic fields in a simple (i.e., linear, isotropic and homogeneous) material medium where a charge distribution of volume charge density ρ and a current distribution of current density $\vec{j}$ exist.

(2). Show that the electromagnetic boundary conditions at the interface S between two regions of simple material media, which we label 1 and 2 are

$$\vec{n} \cdot (\vec{D}_1 - \vec{D}_2)|_S = \rho_S,$$

$$\vec{n} \cdot (\vec{B}_1 - \vec{B}_2)|_S = 0,$$

$$\vec{n} \times (\vec{E}_1 - \vec{E}_2)|_S = 0,$$

and

$$\vec{n} \times (\vec{H}_1 - \vec{H}_2)|_S = \vec{j}_S,$$

where $\vec{n}$ is the normal from region 2 to region 1, ρ_S is the surface charge density, and $\vec{j}_S$ is the surface current density at the interface.

(25%) 4. Consider an elemental oscillating electric dipole (in a simple medium), which consists of a very short conducting wire of length b terminated in two small conductive spheres or disks. Assume the current in the wire to be uniform and to vary sinusoidally with time:

$$I = Re(I_0\, e^{i\omega t}).$$

(1). In the radiation zone, the electric field of this elemental electric dipole is given by

$$\vec{E}(\vec{r}, t) = \hat{\theta}\, \sqrt{\mu/\epsilon}\, \frac{kI_0 b\, sin(kr - \omega t)}{4\pi r}\, sin\theta.$$

Find the associated magnetic field.

(2). Show that the total power radiated by this elemental electric dipole is given by

$$< P >_{Av.} = \frac{1}{12\pi}\sqrt{\mu/\epsilon}\, (kI_0 b)^2.$$

參考用